Positive Self-Image over Time⋆

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Abstract

This paper incorporates egocentric comparisons into a human capital accumulation model and studies the evolution of positive self image over time. The paper shows that the process of human capital accumulation together with egocentric comparisons imply that positive self image of a cohort is first increasing and then decreasing over time. Additionally, the paper finds that positive self image: (1) peaks earlier in activities where skill depreciation is higher, (2) is smaller in activities where the distribution of income is more dispersed, (3) is not a stable characteristic of an individual, and (4) is higher for more patient individuals.

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1 Introduction

Evidence from economics and psychology shows that entrepreneurs, currency traders, fund managers, car drivers and aviation pilots have one thing in common: they all hold overly positive views of their relative skill.[1] The evidence also shows that, contrary to the predictions of rational learning models, positive self image and optimism do not necessarily diminish with experience.[2]

This paper uses a continuous time human capital accumulation model to study the evolution of positive self image over time. Individuals start with heterogeneous stocks of initial skills and have heterogeneous abilities to turn investments in skills into skills. As in Santos-Pinto and Sobel (2005), the paper assumes that skills have different productivity for different individuals and that individuals make egocentric comparisons, that is, when they compare their skills to the skills of others they measure the productivity of others' skills using their own productivity.[3]

The novelty here, by comparison with Santos-Pinto and Sobel (2005), is that the evolution of the stocks of skills over time (and positive self image) is determined by an individual's decision to maximize lifetime disposable income. This decision depends, among other things, on the rate of skill depreciation, the ability to produce human capital, and the rate of time preference, variables whose impact on skill acquisition (and positive self image) were not considered in Santos-Pinto and Sobel (2005).[4]

The main finding of the paper is that the process of human capital accumulation, skill depreciation, and egocentric comparisons imply that positive self image of a cohort is first increasing and then decreasing over time. The driving forces behind this result are: skill investments and egocentric comparisons, skill depreciation, and a finite working life. Consider the start of an cohort's working lifetime before any skill investments are undertaken.[5] If individuals' initial skills,

[1] Following, Santos-Pinto and Sobel (2005), this paper refers to this bias as positive self image. In the psychology literature, positive self image, falls under the rubric of "biases in judgment" together with optimism (overestimation of the chances of experiencing favorable events), overconfidence (overestimation of the precision of forecasts), and the causal attribution bias (the fact that most people tend to attribute success to effort or ability and failure to bad luck). For a detailed discussion of these and other judgment biases see Rabin (1998).

[2] For example, Arabsheibani et al. (2000) find that entrepreneurs' optimism about financial outcomes increases until age 36 before starting to decay. The evidence is discussed in detail in Section 2.

[3] Results from social psychology justify our modeling approach. The evidence demonstrates that individuals make egocentric comparisons when evaluating the abilities of others. That is, in order to evaluate the behavior of others, they apply the standards that they use on themselves. For a review of the evidence see Santos-Pinto and Sobel (2005).

[4] Santos-Pinto and Sobel (2005) show that in the presence of skill enhancement, egocentric comparisons lead to positive self image. Their approach implies that positive self image in a cohort should be increasing with experience provided that skill investment opportunities are increasing with experience. However, Santos-Pinto and Sobel (2005) do not develop a formal model that explains the evolution of positive self image over time.

[5] In the human capital accumulation model used in this paper we do not model the choice between time spent in formal education and time working. Thus, when we talk about working lifetime this includes the formal education period of individuals' lives.

ability to produce human capital and productivity of skills are independently distributed then, on average, there is no positive self image in the population. The assumption that the productivity of skills is heterogeneous across individuals implies that individuals will make different skill investments: each individual will invest more in the skills that he values the most. The assumption that individuals use their own productivity to measure other's skills implies that they will develop, on average, a positive self image. The fact that in the early stages of working life investments in human capital are large implies that positive self image will rise rapidly during that time. When individuals approach the latter stages of their working life new investments in human capital are small and skill depreciation takes over. This reduces the stock of each skill proportionally to its current level which in turn reduces positive self image since individuals have larger stocks of the skills they value the most.[6]

Additionally, the paper presents four new testable implications of the egocentric comparisons and skill acquisition framework. First, positive self image should peak earlier in activities that use skills with high depreciation rates (e.g., information technology jobs) than in activities that use skills with low depreciation rates (e.g., clerical jobs). The intuition for this results is straightforward. If the rate of skill depreciation is excessively high, then the process of human capital accumulation during a finite working lifetime implies that the stock of each skill is increasing during most of an individual's working life and decreasing as an individual's working lifetime approaches the end. The higher the skill depreciation rate, the earlier will the stock of each skill attain it's peak. Since positive self image is highest when individuals' have the largest stocks of skills, the higher the skill depreciation rate the earlier will positive self image attain it's peak.

Second, the model predicts that if there are strong diminishing returns to the production of skills from increases in ability to produce human capital, then one should find smaller levels of positive self image in activities where the distribution of income is more dispersed. The intuition for this result is as follows. It is a well know result from standard human capital accumulation models that an increase in heterogeneity in ability to produce human capital increases income dispersion. The result also applies to our model. Additionally, if there are strong diminishing returns to the production of skills from increases in ability to produce human capital, then an increase in heterogeneity in ability to produce human capital, besides increasing income dispersion, also reduces positive self image. This happens because when individuals' ability to produce human capital becomes more variable the chance of moving up in relative rankings through skill investment decreases.

Third, positive self image should not be a stable characteristic of an individual. For the majority of individuals positive self image should first increase and then decrease over time but for a minority–those who start with high initial

[6] When there is no skill depreciation the paper shows that positive self image of a cohort is always increasing over time. Thus, according to this model, a positive rate of skill depreciation is a necessary condition for obtaining a inverse U-shaped pattern for positive self image over time.

skills and who have low ability to produce human capital–positive self image should decrease over most of their working lifetime.

Fourth and last, more patient individuals should exhibit more positive self image. When individuals are impatient they discount the future heavily and so they will devote fewer resources to producing human capital. If that is the case, then the correlation between productivity and final skills will be smaller and so will be positive self image.

The paper presents two applications of the model. We show how the model can make sense on data on trading activity and trading experience in financial markets. We also show how an extension of the model may explain why poker players' perceptions of relative skill become more inflated over time whereas the perceptions of chess players become more accurate.

The rest of the paper proceeds as follows. Section 2 reviews the empirical evidence. Section 3 introduces the model. Section 4 contains the main findings. Section 5 discusses the main assumptions of the model and additional implications. Section 6 presents two applications. of the model. Section 7 discusses alternative explanations for the relation between positive self image and experience. Section 8 concludes the paper. The Appendix contains the proofs of all results.

2 Empirical Evidence

The tendency that individuals have to make overly positive evaluations of their relative abilities is a staple finding in psychology. According to Myers (1996), a textbook in social psychology: "(...) on nearly any dimension that is both *subjective* and *socially desirable*, most people see themselves as better than average."[7]

Studies on the evolution of individuals' perceptions of relative ability over time can be divided into two types: cross-sectional and longitudinal. In a cross-sectional study a group of individuals with different levels of experience is formed. Each individual within the group is asked to make an evaluation of relative ability. Finally, the group is divided into sub-groups that share a similar level of experience with the task and the level of positive self image of each sub-group is obtained.

In a longitudinal study a group of individuals with the same level of experience with a task is formed. Each individual within the group is asked to make an evaluation of relative ability vis-a-vis the group. The process is repeated

[7] This tendency is also present in individuals' self-assessments of relative performance in their jobs. Myers (1996) cites a study according to which: "In Australia, 86 percent of people rate their job performance as above average, 1 percent as below average." Baker et al. (1998) cite a survey of General Electric Company employees according to which: "58 percent of a sample of white-collar clerical and technical workers rated their own performance as falling within the top 10 percent of their peers in similar jobs, 81 percent rated themselves as falling in the top 20 percent. Only about 1 percent rated themselves below the median." Wichman and Ball (1983) and O'Hare (1990) find that most general aviation pilots believe that they are safer, are less likely to take risks in flight, and possess greater flying skills than their peers.

at different points in time, that is, several evaluations of relative ability are obtained as the group gains more experience with the task.

2.1 Cross-sectional Studies

Road safety institutes in Europe started a consortium to explore car drivers' attitudes to road safety called SARTRE.[8] Among other things, car drivers were asked to assess their relative driving safety and to report their driving experience. The results for each country are summarized in Table 1, constructed from data available in SARTRE 3 (2004a).

Table 1

Mean self-assessments of relative driving safety and driving experience						
Country	Car drivers	0-10	11-20	21-30	31-40	>40
Italy	996	3.89	4.15	4.17	4.20	4.38
Ireland	993	4.11	4.11	4.23	4.27	4.35
Portugal	1017	3.92	4.07	4.24	4.39	4.56
Germany	966	3.79	4.11	4.12	4.14	4.37
Hungary	1014	3.73	3.83	3.88	4.14	4.22
Croatia	1035	3.78	3.98	4.14	4.15	4.60
Estonia	1000	3.62	3.88	3.90	3.91	4.21
U.K.	1218	3.93	3.93	4.03	4.14	4.04
Denmark	1057	3.65	3.77	3.90	3.93	4.08
Switzerland	872	3.72	3.93	3.96	4.11	4.28
Austria	979	3.69	3.87	3.92	4.01	4.00
Slovenia	1056	3.68	3.79	3.90	3.90	4.06
Cyprus	754	3.67	3.79	3.92	4.01	4.86
France	966	3.65	3.74	3.79	3.84	3.90
Poland	1015	3.71	3.91	4.00	4.01	4.13
Slovakia	1111	3.56	3.64	4.10	4.05	3.57
Netherlands	1008	3.66	3.72	3.98	3.86	3.83
Spain	1660	3.70	3.78	3.82	3.86	3.82
Greece	1000	3.72	3.73	3.68	3.95	4.00
Czech Rep.	992	3.66	3.64	3.86	4.02	4.13
Belgium	947	3.41	3.53	3.82	3.77	4.12
Sweden	993	3.60	3.68	3.71	3.71	3.68
Finland	997	3.35	3.55	3.59	3.52	3.50

Table 1 exhibits the mean assessments of drivers in each European country according to driving experience.[9] To construct this table drivers were divided

[8]SARTRE is the abbreviation of Social Attitudes to Road Traffic Risk. In 2002 the SARTRE 3 project was launched and gathered data of car drivers opinions and reported behavior in 23 European countries. In each participating country, a sample of about 1000 car driving license holders, representing the active car drivers' population, have been interviewed face-to-face using a questionnaire.

[9]To assess drivers' perceptions of relative driving safety drivers were asked the following question: "Compared to other drivers, do you think your driving is...........dangerous?" Drivers

into five groups according to driving experience: 0 to 10 years, 11 to 20 years, 21 to 30 years, 31 to 40 years, and more than 40 years driving experience. Table 2 shows us that in 13 out of 23 countries the mean assessments of relative driving safety are increasing with driving experience. In 7 countries there is an inverted U-shaped profile: United Kingdom, Austria, Slovakia, Netherlands, Spain, Sweden, and Finland. For 3 countries–Greece, Czech Republic, and Belgium–there is a positive relation between mean assessments of relative driving safety and driving experience but the relation is neither monotonic nor exhibits an inverted U-shaped profile.[10]

Brozynski et al. (2004) find evidence of positive self image in German fund managers.[11] They also collect data on each fund manager's professional experience. Fund managers were divided into "inexperienced" (less than 5 years of professional experience), "experienced" (more than 5 and less than 15 years of professional experience), and "very experienced" (more than 15 years of professional experience). The mean assessment of the inexperienced group was 2.33, the mean assessment of the experienced group was 2.72, and the mean assessment of the very experienced group was 2.89.

Fraser and Greene (2006) find that British entrepreneurs are more optimistic about their financial outcomes than wage-workers. They also find that optimism of entrepreneurs diminishes with experience in a sample from the 1990s. However, in a sample from the 1980s, entrepreneurs' optimism increased at low experience levels (from 0 to 5 years) before starting to decay.

2.2 Longitudinal Studies

We are not aware of any study that looks at the longitudinal relation between positive self image and experience of a group of individuals. However, some studies are very close to being longitudinal in the sense that they ask two groups, each with a different level of experience with a task, to evaluate relative ability within the group. Two such studies are Wilson and Fallshore (2001) and Glaser et al. (2005).

Wilson and Fallshore's (2001) study aviation pilots' perceptions of relative

could pick one out of five possible answers to fill the gap in the question: "much more" (coded as 1) "a bit more" (coded as 2), "about the same," (coded as 3), "a bit less," (coded as 4), and "a lot less," (coded as 5). Drivers in all countries exhibited a strong tendency to consider their own driving behavior less dangerous than others' driving behavior: the average share of drivers in the 23 European countries who consider themselves to drive less dangerously than others is equal to 63%. On average, only 5% of drivers in all countries consider to drive more dangerously than others.

[10] It is important to bear in mind that the fact that positive self image of car drivers is increasing with driving experience, does not imply that more experienced drivers over estimate their driving ability more than less experienced drivers. In the absence of egocentric comparisions, the human capital accumulation model implies that, on average, more experienced drives should be more skilled than less experienced drivers.

[11] The survey asked: "How do you evaluate your own performance compared to other fund managers?" The fund managers could pick from 5 categories from "much better" (coded as 5) to "much worse" (coded as 1). The mean assessment for all fund managers was 2.67 which indicates a tendency to see oneself as better than others.

flying ability. In this study aviation pilots were asked to report their flight hours and where also asked to assess their relative ability to avoid inadvertent flight into cloud or fog (and to fly out of cloud or fog) by comparison with other pilots with similar flight experience.[12] Wilson and Fallshore (2001) find that flight hours is a significant predictor of pilots' assessments of their relative ability to avoid inadvertent flight into cloud or fog and of their relative ability to fly out of cloud or fog.

Glaser et al. (2005) study the impact of expertise on several judgment biases. To this purpose they run two experiments. The first one involving a group of 29 German professional traders at a bank (median age of 33 years, median of 5 years of experience in the bank, 14 had a university diploma) and a control group of 75 advanced students in Banking and Finance (median age of 24 years). The second one involving a group of 90 professional investment bankers (median age of 34 years) and another control group of 76 advanced students (median age of 24 years). Among other judgment biases, they wanted to compare positive self image of professionals to that of students. They asked subjects, to state subjective confidence intervals for 20 questions (ten questions concerning general knowledge and ten questions concerning economics and finance). After that, each professional was asked to evaluate his own performance and the performance of an average professional. Similarly, each student was asked to evaluate his own performance and the performance of an average student. Glaser et al. (2005) find that in both experiments the degree of positive self image of professional traders is greater than that of the student control group. Thus, the experience of professionals traders seems to exacerbate the degree of positive self image rather than reduce it.

2.3 Studies with Objective Baselines

A few studies, besides asking individuals to provide subjective evaluations of their relative ability, also compare individuals' self-assessments to objective criteria. The objective criteria can be performance at a task or financial outcomes. Thus, in these studies it is possible to have an idea of the degree of over estimation of relative ability of each individual by comparison with an objective baseline.

Arabsheibani et al. (2000) find that British entrepreneurs are more susceptible to expect better financial outcomes than do employees but experience worse realizations. Additionally, they find that entrepreneurs' optimism peaks at age 36 before starting to decay.

[12] In this study one question asked (pp. 2): "In comparison with other pilots with similar flight background and experience as yourself, how would you rate your ability to avoid inadvertent flight into instrument meteorological conditions (i.e., cloud or fog)?" and another question asked, "In comparison with other pilots with similar flight background and experience as yourself, how would you rate your ability to successfully fly out of instrument meteorological conditions should inadvertent flight into cloud or fog occur?" The pilots' answers show that they believed they were more capable than average at avoiding inadvertently flying into cloud or fog and being able to successfully fly out of cloud or fog.

Oberlechner and Osler (2004) use to survey to study positive self image in North American currency market professionals.[13] Almost three quarters of traders (73.6 percent) perceive themselves as more successful than other currency traders. Both traders at top tier and lower tier institutions exhibited the same tendency. A strong tendency for overestimation of relative performance was also found when foreign exchange traders self assessments were compared to their superiors' assessments. The survey participants tended to be fairly experienced and high-ranking.[14] Traders' work experience in the FX market was positively correlated with a positive self-assessment.[15]

Park and Santos-Pinto (2005) find that participants in poker and chess tournaments exhibit overly positive views of their relative ability even when given monetary incentives to make accurate predictions. They also find that overestimation of relative performance of poker players is increasing with an increase in poker players' experience with poker tournaments. By contrast, they find that chess players' forecasts of relative performance in tournaments become more accurate with experience.

3 The Model

Consider the following continuous time human capital accumulation problem, a version of a model that was first analyzed by Ben-Porath (1967):

$$\begin{aligned} \max \quad & \int_0^T \left[\lambda_1 K_1(t) + \lambda_2 K_2(t) - I_1(t) - I_2(t)\right] e^{-\rho t} dt \\ \text{s.t.} \quad & \dot{K}_i(t) = A^{\alpha/2} \left[I_i(t)\right]^b - \delta K_i(t), \ i = 1, 2 \\ & K_i(0) > 0, \ i = 1, 2 \end{aligned} \tag{1}$$

where K_i represents units of skill i, λ_i represents the marginal productivity of skill i, and $I_i(t)$ represents the amount spent to increase skill i.[16]

[13] Among other things the survey asked: "How successful do you see yourself as an FX trader?" The top rank of 7 was assigned to "Much more successful than other FX traders;" the bottom rank of 1 was assigned to "Much less successful than other FX traders." Oberlechner and Osler (2004) also asked participants' immediate superiors (i.e. head traders or chief dealers) to rank them on a seven-point scale for three separate measures of performance: "trading potential," "trading profits," and "overall contribution to the organization." The currency markets professionals gave themselves a mean ranking of 5.06 or "better than average."

[14] The average work experience in the foreign exchange market was 12 years and 75 percent of the participants were senior traders.

[15] Oberlechner and Osler (2004) also found that the difference between a participant's self rating and a composite measure of the superiors' three ratings was positively correlated with rank, that is, individuals who overestimate their relative performance more by comparison with their superiors' assessments have a tendency to have a higher rank in the institution.

[16] The human capital accumulation model introduced by Ben-Porath (1967) has proved one of the most succesfull models in explaining the evolution of individuals' earnings over the lifecycle. The model has stood empirical testing and provides a plausible theoretical benchmark to study skill investment decisions over time.

According to this model an individual chooses how much to invest in each of two skills with the objective of maximizing his discounted sum of disposable income over his life cycle. Disposable income is the difference between gross income and the amount spent in goods and services to increase the two skills, $I_1(t) + I_2(t)$.[17] Gross income is an increasing function of the stock of each skill $K_i(t)$ and its productivity λ_i.[18] More precisely, gross income is a linear function of the two skills weighted by their productivity.

The model assumes that an individual cannot buy skills by going to the capital market, instead he has to produce them. The rate of change of the stock of each skill, $\dot{K}_i(t)$, is determined by the amount that is produced, $A^{\alpha/2}\left[I_i(t)\right]^b$, where $A > 1$, $\alpha \in (0,2)$, and $b \in (0,1)$, less the depreciated stock $\delta K_i(t)$, where δ is the constant rate of depreciation and $\delta \in [0,1]$.[19] The parameter A measures the ability of an individual to produce human capital. The assumption that $\alpha \in (0,2)$ implies that there are decreasing returns to the production of skills from increases in the ability to produce human capital.[20] The parameter b measures the impact of investments in goods and services on skill production.[21] The assumption that $b \in (0,1)$ implies that the production of skills exhibits decreasing returns to increases in direct expenditures in goods and services.[22]

The parameter ρ measures the rate of time preference. We assume that $\rho > 0$. The larger is ρ the more individuals are impatient in the sense that they show a stronger preference for present consumption over future consumption.

3.1 Solving the Model

Applying standard control theory to problem (1) one finds that the evolution of investment in skill i is given by

$$\dot{I}_i(t) = \frac{\rho + \delta}{1-b} I_i(t) - \frac{A^{\alpha/2} b \lambda_i}{1-b}\left[I_i(t)\right]^b, \quad i = 1, 2. \tag{2}$$

[17] We assume that the unit cost of investment in each skill is the same. This assumption is not critical to the results.

[18] The literature on human capital accumulation and lifecycle earnings usually assumes that there is only one skill and that earnings are a linear function of that skill. This assumption simplifies the algebra substantially.

[19] Different depreciation rates for each skill can be introduced, but this generalization has no implications in terms of the main results of the model.

[20] Usually, the production function would be specified with two inputs: current skill stock and the amount spend in market goods. Assuming that the production of skills also depends on current skill levels complicates the algebra without changing the main insights in the paper. In some models of human capital accumulation individuals also have to choose how much time to devote to market production versus skill production. We abstract from this choice to make the analysis clearer.

[21] We could have allowed for $\alpha_1 \neq \alpha_2$ and $b_1 \neq b_2$, and also for different prices of expenditures in goods and sevices in each skill. This generalization also has no implications in terms of the main results of the model. We deliberately choose to assume symmmetry in the cost and production of skills to focus on the implications of heterogeneity in skill productivity in terms of skill investments.

[22] In this model initial stocks of skills are pure rents. Individuals with higher inital skills tend to have higher lifetime earnings, but they do not make larger investments in skill production.

Equation (2) is a Bernoulli differential equation with constant coefficients with solution given by

$$I_i(t) = \left(\frac{A^{\alpha/2} b \lambda_i}{\rho+\delta}\right)^{\frac{1}{1-b}} \left(1 - e^{-(\rho+\delta)(T-t)}\right)^{\frac{1}{1-b}}, \quad i = 1, 2. \tag{3}$$

From (3) we see that in this model the amount invested in skills is decreasing over time reaching zero at $t = T$. At the beginning of an individual's working lifetime there are strong incentives to produce human capital since at that time human capital generates income for many periods. Similarly, when an individual approaches the end of his working life there are almost no incentives to produce new human capital since at that time human capital only generates income for very few periods. We also see from (3) that investment in skills does not depend on the stocks of skills.[23]

Substituting (3) into the equation of the evolution of the stock of skill i

$$\dot{K}_i(t) = A^{\alpha/2} \left[I_i(t)\right]^b - \delta K_i(t) \quad i = 1, 2,$$

gives us

$$\dot{K}_i(t) = A^{\alpha/2} \left(\frac{A^{\alpha/2} b \lambda_i}{\rho+\delta}\right)^{\frac{b}{1-b}} \left(1 - e^{-(\rho+\delta)(T-t)}\right)^{\frac{b}{1-b}} - \delta K_i(t) \quad i = 1, 2. \tag{4}$$

We see from (4) that at the end of an individual's working lifetime we have that $\dot{K}_i(T) = -\delta K_i(T)$, that is, since there is no new production of human capital at time T, the stock of each skill must be reduced by the amount of depreciation.

From now on it will be assumed that $b = 1/2$. This assumption makes the problem easier without making the results in any way less general.[24] Thus, setting $b = 1/2$ in (4) we obtain

$$\dot{K}_i(t) = \frac{1}{2} \frac{A^\alpha \lambda_i}{\rho+\delta} \left(1 - e^{-(\rho+\delta)(T-t)}\right) - \delta K_i(t), \quad i = 1, 2. \tag{5}$$

Equation (5) is a linear nonhomogeneous differential equation with solution given by

$$K_i(t) = K_i(0) e^{-\delta t} + A^\alpha \lambda_i \omega(t) \tag{6}$$

where

$$\omega(t) = \frac{1}{2\delta(\rho+\delta)} \left[1 - e^{-\delta t} - \frac{\delta e^{-(\rho+\delta)(T-t)}}{\rho+2\delta} \left(1 - e^{-(\rho+2\delta)t}\right)\right]. \tag{7}$$

[23]This happens because the production function of human capital does not depend on curent skills levels.

[24]Equation (4) can be solved for any real number contained in $(0, 1)$. When $b/(1-b)$ is an integer, the solution to (4) is a finite series. When $b/(1-b)$ is not an integer, the solution to (4) is an infinite series. For a detailed discussion of this simplifying assumption see Haley (1973) and Haley (1976).

Equation (6) describes the evolution of the stock of skill i given the initial stock of that skill, the rate of human capital depreciation, the ability to produce human capital, the productivity of the skill, and the rate of time preference.[25] From (6) we see that if an individual's initial stocks of each skill are identical, then he will have more of the skill that is more valuable to him.

Understanding the behavior of the function $\omega(t)$ will be critical for understanding the evolution of positive self image over time. Thus, our first result characterizes the function $\omega(t)$.

Lemma 1 *The function $\omega(t)$ verifies four properties: (i) $\omega(0) = 0$, (ii) $\omega(T) > 0$, (iii) $\omega(t)$ is concave, and (iv) $\omega(t)$ attains its maximum at t^*, with $t^* \in (0, T)$.*

Lemma 1 characterizes the behavior of the stocks of skills over time not taking into account the impact of depreciation of initial skills. This result tells us that skill depreciation together with a finite working lifetime imply that: (a) when the stock of each skill is increasing in the beginning of an individual's working life then it must decrease as an individual's working lifetime approaches the end, (b) when the stock of each skill is decreasing in the beginning of an individual's working life then it must decrease faster as an individual's working lifetime approaches the end.[26]

3.2 Skill Comparisons

Assume that initial skills $K_i(0)$, $i = 1, 2$, ability to produce human capital, A, and productivity of skills, λ, are independently distributed. Let $\lambda_1 = \lambda$ and $\lambda_2 = 1 - \lambda$ and assume that λ has a symmetric Beta distribution.[27] Finally, assume that A has a distribution with support on $\left[\underline{A}, \bar{A}\right]$ with $1 \leq \underline{A} < \bar{A}$ and that initial skills have a distribution with support on R^+.

An individual with initial skills $K(0)$, ability to produce skills A, and productivity of skills λ, measures his ability at time t as

$$W^*(t; K(0), A, \lambda) = W(\phi(t; K(0), A, \lambda), \lambda) = \lambda K_1(t) + (1 - \lambda) K_2(t), \qquad (8)$$

where $\phi(t; K(0), A, \lambda)$ denotes the optimal stocks of skills at time t as a function of parameters $K(0), A$, and λ. Making use of (6) ones has that

$$\begin{aligned} W^*(t; K(0), A, \lambda) &= [\lambda K_1(0) + (1 - \lambda) K_2(0)] e^{-\delta t} \\ &\quad + A^\alpha \left[\lambda^2 + (1 - \lambda)^2\right] \omega(t). \end{aligned}$$

[25] We make no assumptions on the parameters of the model to condition the behavior of the stock of skill i over time since this is not relevant to our results on positive self image. However, a natural assumption in a human capital accumulation model with finite time would be that the parameters of the model are such that the stock of skills are increasing in the beginning and middle of a individual's working life and decreasing towards the end of an individual's working life.

[26] The second situation can happen in professions where the ability to produce human capital is very low and initial talent is almost all that matters. All the findings in the paper also apply to this case.

[27] The results in the paper are valid for more general distributions for λ.

An individual with initial skills $K(0)$, ability to produce skills A, and productivity of skills λ, measures the expected ability of the population at time t as

$$E_{(K'(0),A',\lambda')}\left\{W(\phi(t;K'(0),A',\lambda'),\lambda)\right\} = \lambda\bar{K}_1(t) + (1-\lambda)\,\bar{K}_2(t), \tag{9}$$

where $\bar{K}_i(t)$, $i = 1,2.$ denote the average skill levels in the population at time t. Making use of (6) one has that

$$E_{(K'(0),A',\lambda')}\left\{W(\phi(t;K'(0),A',\lambda'),\lambda)\right\}$$
$$= \left[\lambda\bar{K}_1(0) + (1-\lambda)\bar{K}_2(0)\right]e^{-\delta t} + E(A^{\alpha})\frac{1}{2}\omega(t),$$

and $\bar{K}_i(0)$, $i = 1,2$, denote the average initial skills in the population. Following Santos-Pinto and Sobel (2005), let

$$D^*(t;K(0),A,\lambda) = W^*(t;K(0),A,\lambda) - E_{(K'(0),A',\lambda')}\left\{W(\phi(t;K'(0),A',\lambda'),\lambda)\right\} \tag{10}$$

be the difference between an individual's ability and the expected ability of the population, where ability is measured according to that individual's productivity. Refer to $D^*(t;K(0),A,\lambda)$ an individual's **ability gap** at time t.

Substituting (8) and (9) into (10) gives us

$$\begin{aligned} D^*(t;K(0),A,\lambda) \quad &= \quad \lambda\left[K_1(0) - \bar{K}_1(0)\right]e^{-\delta t} + (1-\lambda)\left[K_2(0) - \bar{K}_2(0)\right]e^{-\delta t} \\ &\quad + \left\{A^{\alpha}\left[\lambda^2 + (1-\lambda^2)\right] - E(A^{\alpha})\frac{1}{2}\right\}\omega\left(t\right) \end{aligned} \tag{11}$$

It follows directly from (i), (ii), and (iii) in Lemma 1 that $\omega(t) > 0$ for $t \in (0,T]$. This implies that an individual's ability gap at time t is increasing in A. The ability gap is always positive for individuals who have high initial skills and who have high ability to produce human capital. The ability gap can be negative for individuals who have low ability to produce human capital.

Since initial skills $K_i(0)$, $i = 1,2$, ability to produce human capital, A, and productivity of skills, λ, are independently distributed we have that the **expected ability gap** of a cohort at time t is equal to

$$E_{(K(0),A,\lambda)}D^*(t;K(0),A,\lambda) = 2E\left(\lambda - .5\right)^2 E(A^{\alpha})\omega\left(t\right).$$

The expected ability gap is positive for all $t \in (0,T]$ since $E\left(\lambda - .5\right)^2 > 0$, $E(A^{\alpha}) > 0$, and $\omega(t) > 0$ for $t \in (0,T]$. Thus, the cohort exhibits a positive self image during the entire working lifetime.

4 Results

The main result of the paper describes the pattern of positive self image over time implied by the human capital accumulation and egocentric comparisons model when there is a positive rate of skill depreciation.

Proposition 1 *If $\delta \in (0, 1]$, then the expected ability gap is increasing with t for $0 < t < t^*$ and decreasing with t for $t^* < t < T$, where $t^* = \arg\max \omega(t)$.*

Proposition 1 tells us if skills depreciate, then human capital accumulation and egocentric comparisons imply that a cohort's positive self image increases at the beginning of working lifetime, reaches it's peak at t^*, and then decreases until the end of working lifetime. Since the intuition for this result was already discussed in Section 1 let us now discuss the main assumptions behind it.

Clearly, the assumption of heterogeneity in skill productivity together with the assumption that individuals make egocentric comparisons are the ones that are responsible for an increase in positive self image in the earlier stages of working life. Support for these assumptions can be found in Santos-Pinto and Sobel (2005) and will not be discussed here.

Let us then discuss the role of the assumption of positive skill depreciation. We can show that if there is no skill depreciation then positive self image, measured by the expected ability gap, is always increasing over time. To see this notice that positive self image reaches it's peak at t^*, where $t^* = \arg\max \omega(t)$. From the definition of $\omega(t)$ and Lemma 1 we know that t^* is the solution to

$$e^{-\delta t} - \frac{\delta}{\rho + 2\delta} e^{-(\rho+\delta)T - \delta t} - \frac{\rho + \delta}{\rho + 2\delta} e^{-(\rho+\delta)(T-t)} = 0. \tag{12}$$

Solving (12) for t we have that

$$t^* = \frac{\ln\left[(\rho + 2\delta) e^{(\rho+\delta)T} - \delta\right] - \ln(\rho + \delta)}{\rho + 2\delta} \tag{13}$$

If we set $\delta = 0$ in (13) then $t^* = T$. Thus, if human capital does not depreciate, then positive self image of a cohort is always increasing over time.

Taking a linear approximation of t^* around $\delta = 0$ we have that

$$t^* \approx \left(\frac{\rho - \delta}{\rho}\right) T + \left(\frac{1 - e^{-\rho T}}{\rho^2}\right) \delta. \tag{14}$$

By inspection of (14) we can also see that if the rate of time preference is close to one, then $\frac{\rho-\delta}{\rho}T$ is a good approximation to t^*.[28] Thus, if the rate of time preference is close to one and the rate of skill depreciation is close to zero, then positive self image of a cohort reaches it's peak close to the end of working life.[29]

The approximation also shows that positive self image should peak earlier in activities where skill depreciation is high (e.g., computer programming, playing a musical instrument) than in activities where skill depreciation is low (e.g., typing, sorting and flipping through files) since $\frac{\rho-\delta}{\rho}T$ is decreasing with δ.

[28] In fact, when models of human capital accumulation are calibrated to match the empirical facts on the evolution of the earnings distribution of a cohort over time the rate of time preference is usually chosen to be larger than .75 and the rate of skill depreciation to be smaller than .25. See for example Huggett et al. (2002).

[29] Simulations of the model with different parameter values confirm this. For example, with $T = 60$, $\rho = .8$, and $\delta = .1$, we have that $t^* = 54.105$. The approximation gives us $\frac{\rho-\delta}{\rho}T = \frac{.7}{.8}60 = 52.5$.

Another implication of the model is stated formally in the next proposition.[30]

Proposition 2 *If $\alpha \in (0,1)$, then a mean preserving spread in the distribution of A reduces the expected ability gap for all t.*

Proposition 2 tells us that, everything else constant, an increase in heterogeneity in individuals' ability to produce human capital lowers positive self image at any point in time. This happens because by making individuals' ability to produce human capital more variable the chance of moving up in relative rankings through skill investment decreases. This result is the equivalent of Proposition 9 in Santos-Pinto and Sobel (2005). The novelty here is the interpretation of the result in the context of a human capital accumulation model.

Before discussing the interpretation of the result let us first discuss the assumption that $\alpha \in (0,1)$. This assumption tells us that there are strong diminishing returns to the production of skills from increases in the ability to produce human capital. It guarantees that the expected ability gap is a concave function ability to produce human capital and this implies that an increase in variability in the distribution of A reduces the expected ability gap.[31] It is an empirical matter whether the assumption that $\alpha \in (0,1)$ makes sense or not. At least one paper in the human capital accumulation literature supports this assumption: Kuruşcu (2002).[32]

Taking the assumption that $\alpha \in (0,1)$ at face value let us now turn to the interpretation of Proposition 2. Several papers show that differences in ability to produce human capital are key for human capital accumulation models to be able to explain the evolution of earnings over the life-cycle.[33] In fact, labor economists who use human capital accumulation models to explain the evolution of earnings over the life-cycle agree that the assumption that individuals have different abilities to produce human is the only way to explain the increase in earnings dispersion over the life-cycle.[34]

Proposition 2 shows that heterogeneity in ability to produce human capital constrains the degree of positive self image. More interestingly, this result tells us that, everything else equal, overestimation of relative ability should be smaller in activities where the distribution of income is more dispersed. In other words, controlling for all other variables that have an impact on positive self image (average income, the number of skills required in different activities, experience,

[30] The comparative static results obtained in Santos-Pinto and Sobel (2004) also apply to this model and will not discussed here. For example, positive self image is larger the more variable are productivies of skills in the population, positive self image is larger with one adds a positive constant to the distribution of A, and positive self image relative to an objective measure of ability is decreasing with an increase in an individual's ranking under that objective measure of ability.

[31] If there are weak diminishing returns to the production of skills from increases in the ability to produce human capital, $\alpha \in (1,2)$, then the opposite result would follow, that is, a mean preserving spread in the distribution of A increases the expected ability gap for all t.

[32] See Kuruşcu (2002), pages 22-24 and Table 3 on page 25.

[33] According to Hugget et al. (2002): "(..) mean earnings and measures of earnings dispersion and skewness all increase in US data over most of the working life-cycle for a typical cohort as the cohort ages."

[34] For a good discussion on this topic see Neal and Rosen (1999).

etc.) we should expect to find smaller levels of positive self image if we ask individuals to evaluate their skills in activities where the distribution of income is more dispersed. One implication of this result is that if positive self image leads to poor decision making, then this effect will be small in activities where income is very dispersed but large in activities were income is not very dispersed. For example, Cross (1977) finds that 94% of college instructors think their teaching ability is above average. If college instructors' income does not become dispersed over the life-cycle then the model implies that their high level of positive self image will persist. If college instructors' positive self image leads them to make lower investments in teaching skills, then there can be adverse welfare consequences.[35]

Another implication of the model is that positive self image should not be a stable characteristic of an individual, rather it should be a variable one.[36] As we have seen, the model shows us that the process of human capital accumulation together with egocentric comparisons imply that, on average, a population will gradually develop positive self image over time. However, this does not imply that all individuals in the population will display an increasing positive self image over time. Proposition 1 tells us if the depreciation rate is positive, then for the majority of individuals in a cohort, positive self image should first increase and then decrease over time. However, for a minority, positive self image should decrease over most of working lifetime. This is stated precisely in the next result.

Proposition 3 *If individual λ, A, and $K(0)$ is such that (i) $K_i(0) \geq \bar{K}_i(0)$, $i = 1, 2$ and (ii) $A^{\alpha}\left[\lambda^2 + (1-\lambda^2)\right] - E(A^{\alpha})\frac{1}{2} < 0$, then the ability gap of this individual is decreasing with t for all $t \in (0, t^*)$, where $t^* = \arg\max \omega(t)$.*

Proposition 3 tells us that individuals who are initially very talented but who have low ability to produce human capital will exhibit a decreasing positive self image over time for most of their working lifetime. This result would also be valid in a model without egocentric comparisons. The individuals who are less able to increase their human capital and start with high initial skills will become objectively less skilled by comparison with the population.

We can state one additional result.

Proposition 4 *An increase in ρ reduces the expected ability gap for all t.*

Recall that ρ measures the rate of time preference. If individuals are impatient (ρ is large) they discount the future heavily and so they should devote fewer resources to producing human capital. If that is the case, then the correlation between productivity and final skills will be smaller and so will be the degree of positive self image.

[35] As far as we know there is no empirical evidence as to whether positive self image leads an individual to put more or less effort into tasks or to make larger or smaller investments in skills.

[36] Interestingly, cross sectionally, positive self image may be a stable characteristic of a population in an overlapping generations framework, where the young enter the population pool and the old abandon it.

5 Discussion and Additional Implications

This section discusses the implications of relaxing the main assumptions of the model. It also describes additional implications of the model that could distinguish it from alternative descriptions of behavior.

5.1 Main Assumptions

To better understand the predictions of the model let us consider the implications of dropping its two main assumptions–skill acquisition and egocentric comparisons–one at a time. Suppose first that individuals can not increase their skills but make egocentric comparisons. Since by assumption initial skills and productivity of skills are independently distributed then, on average, individuals should have an accurate view of their relative ability. It also follows that each individual's self image does not change over time either longitudinally or cross-sectionally.

Now, suppose that individuals do not make egocentric comparisons but they are able to increase their skills. If this is the case then all individuals should have an accurate view of his relative ability. Longitudinally, self image will not change over time. However, cross-sectionally, self image will vary with experience. If there is no skill depreciation, then self image should be increasing with experience since, through human capital accumulation, more experienced individuals will have more skills than less experienced individuals.[37]

The model in this paper also assumes that skills have different productivities for different individuals. It would be absurd to pretend that this assumption applies to all settings. It does not. In many activities each skill has the same productivity for all individuals. Even if that is the case we cannot rule out the influence of egocentric comparisons and skill investment in determining individuals' perceptions of relative skill. In fact, it is possible to incorporate skill investment and egocentric comparisons into a Bayesian learning model where each skill has the same productivity across all individuals. For example, one could assume that the process that generates income as a function of skills is given by

$$Y(t) = \sum \lambda_j K_j(t) + \varepsilon(t),$$

where λ_j, $j = 1, ..., J$, represents the productivity of skill j and $\varepsilon(t)$ is a random term. Individuals start with subjective prior beliefs about productivity of skills and learn about the true productivity over time. In this case individual i's perception of the process that generates income would be given by

$$Y^i(t) = \sum \lambda_j^i(t)\, K_j(t) + \varepsilon(t),$$

where $\lambda_j^i(t)$, $j = 1, ..., J$, is the expected productivity of skill j from the perspective of individual i, a function of past observations of income of individual

[37] If there is skill depreciation we know that, cross-sectionaly, positive self image will exhibit a inverted U-shape.

i. In this model individuals choose investments in skills to maximize the sum of their discounted disposable income over the lifecycle. Individuals observe their own income at each period in time and use that information to update their beliefs about the productivity of skills. After updating their beliefs about the productivity of skills individuals use their own beliefs to compare their skills to the skills of others.[38]

In a model like this individuals will use skill investments to learn about the technology, that is, there is learning by experimentation. This complicates the analysis substantially. The pattern of positive self image over time will depend critically on the variability of the random term. If the random term has a large variance, then learning about λ will take time and the impact of skill investment plus egocentric comparisons will persist. In this case positive self image will increase with experience over most of an individual's lifetime. By contrast, if the random term has a small variance, then learning about λ is fast and the impact of skill investment plus egocentric comparisons will vanish quite rapidly. In this case positive self image will decrease with experience over most of an individual's lifetime.

Finally, the model assumes that individuals do not use any empirical observations about the income of their peers to make comparisons. This assumption is not valid for activities where individuals receive unambiguous information about the income of their peers.

5.2 Longitudinal Studies

The model predicts that the longitudinal relation between positive self image and experience depends on the rate of human capital depreciation. If the rate of human capital depreciation is zero individuals' skills are always increasing over time and therefore, longitudinally, positive self image should be increasing with experience. If the human capital depreciation rate is positive, then, longitudinally, positive self image is first increasing and then decreasing with experience.[39]

5.3 Cross-Sectional Studies

The model also generates testable implications regarding the cross-sectional relation between positive self image and experience. If the human capital depreciation rate is zero individuals' skills are always increasing over time and therefore, cross-sectionally, positive self image should be increasing with experience. In this case, there are two forces that lead more experienced individuals to have a more positive view of their relative skill by comparison with less experienced individuals. As time passes more experienced individuals have relatively

[38] We assume that individuals only observe their own income and have no information about the income of their peers. If individuals had full information about the income of their peers, then they could use that information and comparisons would no longer be egocentric.

[39] This result is valid no matter if individuals' skills are first increasing and then decreasing with experience or if they are always decreasing with experience.

more of the skills they value the most but they also have more of all skills by comparison with less experienced individuals. If the human capital depreciation rate is positive and individuals' skills are first increasing and then decreasing over time, then a cross-sectional analysis of positive self image over time should exhibit an inverted U-shaped profile.[40]

5.4 Studies with Objective Baselines

If one assumes that there exists an objective technology, the model in this paper can shed some light on the relation between self image relative to an objective baseline and experience with a task. Here, as before, we need to distinguish between longitudinal and cross-sectional predictions.

The implications of the model regarding the longitudinal relation between self image relative to an objective benchmark and experience are straightforward. We already know that if the rate of human capital depreciation is positive, then, longitudinally, positive self image is first increasing and then decreasing with experience. Since, objectively only 50% of individuals can be above the median at any point in time, then, longitudinally, if the rate of human capital depreciation is positive then self image relative to an objective baseline should be first increasing and then decreasing over time.[41]

The implications of the model regarding the cross-sectional relation between self image relative to an objective benchmark and experience are more complex. To see this consider only the case where the rate of human capital depreciation is zero. In this case we already know that, cross-sectionally, more experienced individuals should exhibit more positive self image than less experienced individuals. We also know that if individuals' skills are increasing with experience, then, on average, more experienced individuals' objective relative performance should be greater than that of less experienced individuals. Now, who over estimates relative ability more by comparison with the objective baseline: the inexperienced or the experienced individuals? The model tells us that the answer will depend on the slopes of the positive self image-experience curve and of the objective baseline-experience curve.[42] If slope of the positive self image-

[40] If the human capital depreciation rate is positive and individuals' skills are decreasing over time, then more experienced individuals will have relatively more of the skills that they value the most but they also have less of all skills by comparison with less experienced individuals. In this case a cross-sectional analysis of positive self image over time can generate three different profiles. If the egocentric comparisons plus skill investment effect dominates, then a cross-sectional analysis of positive self image over time should exhibit an inverted U-shaped profile. If the skill depreciation effect dominates, then, cross-sectionally, positive self image should be decreasing with experience. If the two effects balance out then, cross-sectionally, experience and positive self image should be uncorrelated.

[41] If the rate of human capital depreciation is zero then, longitudinally, positive self image should be increasing with experience.

[42] The slope of the objective baseline-experience curve measures how much ability is correlated with experience. If the slope is positive and small that means that abillity is weakly correlated with experience. This may be the case in professions where innate skills matter a lot and where there are very few opportunities to increase skills. If the slope is positive and large that means that abillity is strongly correlated with experience.

experience curve is greater than the slope of the objective baseline-experience curve then, on average, more experienced individuals are more likely to over estimate their relative ability by comparison with an objective baseline than less experienced individuals. If the slope of the positive self image-experience curve is smaller than the slope of the objective baseline-experience curve then the reverse happens. Finally, if the slope of the two curves is the same, then more and less experienced individuals are, on average, equally likely to over estimate their relative ability by comparison with an objective baseline.

6 Applications

Positive self image may influence behavior in many economically relevant situations. For example, Camerer and Lovallo (1999) find that there is more entry when relative skill determines payoffs, which suggests that individuals overestimated their ability to do well relative to others.

We will now show how the model can be used to make sense of data on trading activity and trading experience in financial markets. We will also show how incorporating egocentric comparisons and skill investment into a rational learning model with subjective prior beliefs about the productivity of skills can explain the patterns of positive self image of poker and chess players.

6.1 Trading Experience and Trading Activity

In the context of financial markets, positive self-image and overconfidence are two of the most prominent explanations for why some individuals trade more frequently more than others. In fact, theoretical models of financial markets predict that positive self image and overconfidence lead to increased trading activity.[43]

We will now show how the model in this paper can be shed light on the question of gender and trading activity, which has been the focus of a number of studies starting with Barber and Odean (2001). The argument in this paper is the following. Theoretical models in finance predict that overconfident investors trade more than rational investors.[44] So, if men are more overconfident than women, then men should trade more than women. Barber and Odean (2001) analyze the common stock investments of men and women from 1991 to 1997 using account data for over 35,000 households from a large discount brokerage and find that men trade 45 % more than women.

The human capital accumulation plus egocentric comparisons model offers an alternative explanation for why men trade more than women in Barber and

[43] Deaves et al. (2003) confirm this prediction using an asset market experiment. Glaser and Weber (2003) also find that investors who think they are better than average in terms of investment skills or past performance trade more.

[44] Here, according to Barber and Odean (2001) overconfident investors means investors who overestimate the precision of their information.

Odean's (2001) data. Suppose that men and women are equally likely to overestimate their relative trading skill, that trading experience increases overestimation of relative skill, and that overestimation of trading skill leads to increased trading activity. If that is the case, then if men have more trading experience than women, then men should trade more than women. In fact, according to Barber and Odean's (2001, pp. 269): "The differences in self-reported experience by gender are quite large. In general, women report having less investment experience than men." More recently, Deaves et al. (2003) have produced experimental evidence that overconfidence leads to increased trading activity, but that positive self image may play a role as well.[45] Additionally, they find that individuals with more trading experience tend to trade more [46]

Barber and Odean (2002) find that the switch from phone-based trading to online trading activity is associated with greater trading activity.[47] They argue that investors who switch to online trading are likely to be more overconfident after going online than before. This happens because these investors usually experience unusually strong performance prior to the switch and low performance after. According to Barber and Odean (2002), the strong performance prior to the leads to overconfidence via the self-serving attribution bias.

The human capital accumulation plus egocentric comparisons model offers an alternative explanation for this finding. Suppose that trading experience increases overestimation of trading skill and that overestimation of trading skill leads to increases in trading activity. If this is the case, then if online investors have more trading experience than other investors, then online investors should trade more. In fact, in Barber and Odean's (2002) data, online investors report having more trading experience than other investors.

6.2 Poker Players and Chess Players

Park and Santos-Pinto (2005) find that overestimation of relative performance of poker players is increasing with experience whereas chess players' forecasts of relative performance become more accurate with experience. If poker is an activity where random factors are very important in determining outcomes, poker players can improve different skills, and make egocentric comparisons, then it may take a long time until experience with poker tournaments reduces poker players' positive self image. By contrast, if chess is an activity where random factors are not so important in determining outcomes, chess players can improve different skills, and make egocentric comparisons, then maybe playing a few chess tournaments is enough to reduce chess players' positive views about their relative skill.

[45] This is in contrast to Glaser and Weber (2003) who find that positive self image is a useful determinant of trading activity whereas overconfidence is not.

[46] In Deaves et al.'s (2003) experiment women have about the same level of both overconfidence and trading activity as do men. Thus, contrary to the findings of Barber and Odean (2001), there is little evidence that overconfidence and trading activity are in any meaninful way related to gender.

[47] There was a dramatic erosion in the performance of online investors after they switch to online trading.

7 Alternative Explanations

There are alternative explanations for positive self image that can account for some of the empirical results discussed in this paper. These alternative explanations do not require that individuals are able to increase their skills. They also do not rely on individuals making egocentric comparisons.

Consider a situation where individuals differ in their ability at a task. To make things simple suppose that individuals can either be of high or low ability and where there is a selection effect that rewards high ability: for example, the high ability individuals survive with probability 75% and the low ability individuals only survive with probability 25%. Furthermore, suppose that every time an individual is wiped out he is replaced by an (inexperienced) individual (who may be of high or low ability with 50% probability each). In this case, the more experienced individuals have, on average, higher ability than the less experienced individuals. Thus, cross-sectionally self image is increasing with experience. It is easy to see that, without any added feature, this description of behavior implies that there is no positive self image in the population. One simple way to generate positive self image is to assume that the individuals who survive are comparing themselves against the wrong pool.[48] For example, experienced individuals may over estimate the percentage of inexperienced individuals in the population. If that is the case and assuming that inexperienced individuals compare themselves against the correct pool, then, on average, individuals will have a positive self image of their relative ability and, cross-sectionally, positive self image will increase with experience.[49,50]

Another alternative explanation is that positive self image causes experience. This happens if positive self image leads to better relative performance and better relative performance (through a selection effect) leads to more experience. For example, positive self-image may lead to better relative performance if it reduces stress.[51] Positive self-image may also lead to better relative performance it has strategic effects on others' that are beneficial to the self.[52] Alternatively, a person with a positive self image may look more aggressive to competitors

[48] This possibility was suggested by Joel Sobel.

[49] If individual who survive have an accurate assessment of the composition of the population and the inexperienced individuals underestimate the percentage of experienced individuals in the population then, there would still be positive self image in the population but, cross-sectionally, positive self image would be decreasing with experience.

[50] If there are strong selection effects towards the survival of the best mutual fund managers or foreign exchange traders, then this explanation can account for the cross-sectional pattern of positive self image displayed by these individuals. However, this is not a convincing explanation for the cross-sectional pattern of positive self image displayed by car drivers. Our own personal experience tells us that the selection effect in driving is either absent or very weak. A very bad driver is much more likely to get into a serious accident and either die or become permanentely injured and unable to drive. However, this is a low probability event and therefore it only affects few bad drivers.

[51] It has been documented that most decision makers have a tendency to make worse decisions under stressful conditions. This possibility is modeled in Compte and Postlewaite (2001).

[52] For example, a person with a positive self image may cause a more favorable impression on his superiors and so may be promoted more quickly.

and this may give that person a strategic hedge.[53] Each of the variations of this second explanation may account for the cross-sectional pattern of positive self image displayed by mutual fund and foreign exchange traders. However, they can not explain the cross-sectional pattern of positive self image of car drivers' since both the selection and the strategic effects are absent.[54]

Finally, experience may cause positive self image through the self-serving bias in causal attributions: attributing good outcomes to ability and bad outcomes to luck.[55] Suppose that, before engaging in a job, individuals have incomplete information about their ability but they know that can be of either high or low ability. Individuals learn about their ability over time by observing a series of experiments that are correlated with ability. If this is the case then, on average, inexperienced individuals will develop a positive self image of their abilities. However, as experience with the task accumulates and provided that individuals are not too biased, they will eventually learn their true ability. In other words, when the self serving bias is not too large the model predicts that, both longitudinally as well as cross-sectionally, positive self image is first increasing and then decreasing with experience. Of course, if the self serving bias is very large then positive self image is always increasing with experience.[56]

8 Conclusion

Rational learning predicts that individuals' beliefs should become more accurate with experience. Much of the empirical evidence on the evolution of positive self image over time reviewed in this paper is at odds with rational learning.

This paper shows that the process of human capital accumulation in the presence of skill depreciation and egocentric comparisons imply that individuals' perceptions of skill do not have to become more accurate over time, on the contrary, they may become increasingly inflated.

We view this explanation as an additional contribution to the literature that studies the evolution of individual perceptions of skill. The results were obtained making strong assumptions. By dropping some of the assumptions the results no longer hold.

An explanation of the evolution positive self image over time across different tasks is beyond the scope of this paper and is left for future research. Still, the paper shows that some of the ingredients that should be part of such an analysis are: (1) the possibility of self-selection into an activity, (2) the presence or absence of skill investment opportunities, (3) the possibility of making egocentric comparisons, and (4) the frequency and quality of information about an individual's performance at the activity.

[53] This approach is modeled in Heifetz and Spiegel (2001).

[54] This explanation is also not able to account for the longitudinal pattern of positive self image displayed by airplane pilots.

[55] This explanation was first modeled by Gervais and Odean (2001).

[56] Note that since this explanation does not require any selection effect, it can also account for the cross-sectional relation between positive self image and driving experience observed in European car drivers.

9 Appendix

Derivation of Equation (2) The Hamiltonian for the human capital accumulation problem is given by

$$\begin{aligned} H &= \left[\lambda_1 K_1(t) + \lambda_2 K_2(t) - I_1(t) - I_2(t)\right] e^{-\rho t} \\ &\quad + \mu_1(t)\left[A^{\alpha/2}\left(I_1(t)\right)^b - \delta K_1(t)\right] + \mu_2(t)\left[A^{\alpha/2}\left(I_2(t)\right)^b - \delta K_2(t)\right]. \end{aligned}$$

The optimality conditions for the control variables are given by

$$\frac{\partial H}{\partial I_i(t)} = -e^{-\rho t} + \mu_i(t) A^{\alpha/2} b \left(I_i(t)\right)^{b-1} = 0, \quad i = 1, 2, \tag{15}$$

and, for the state variables, by

$$\frac{\partial H}{\partial K_i(t)} = \lambda_i e^{-\rho t} - \mu_i(t)\delta = -\frac{\partial \mu_i(t)}{\partial t}, \quad i = 1, 2. \tag{16}$$

Solving (15) for $\mu_i(t)$ and taking logs gives us

$$\ln \mu_i(t) = -\ln A^{\alpha/2} b + (1-b)\ln I_i(t) - \rho t.$$

Taking the derivative with respect to t we have

$$\frac{\partial \ln \mu_i(t)}{\partial t} = (1-b)\frac{\partial \ln I_i(t)}{\partial t} - \rho,$$

or

$$-\frac{\partial \mu_i(t)}{\partial t}\frac{1}{\mu_i(t)} = -(1-b)\frac{\partial I_i(t)}{\partial t}\frac{1}{I_i(t)} + \rho.$$

Making use of (15) and (16) we have that

$$\left[\lambda_i \mu_i(t) A^{\alpha/2} b \left(I_i(t)\right)^{b-1} - \mu_i(t)\delta\right]\frac{1}{\mu_i(t)} = -(1-b)\frac{\partial I_i(t)}{\partial t}\frac{1}{I_i(t)} + \rho,$$

which after simplification gives us

$$\frac{\partial I_i(t)}{\partial t} = \frac{\rho+\delta}{1-b} I_i(t) - \frac{\lambda_i A^{\alpha/2} b}{1-b}\left(I_i(t)\right)^b, \quad i = 1, 2.$$

which is equation (2). *Q.E.D.*

Derivation of Equation (3) Equation (2) is a Bernoulli differential equation with constant coefficients and can be solved by performing a change of variable. If we let $W_i(t) = \left(I_i(t)\right)^{1-b}$ we have that

$$\frac{\partial I_i(t)}{\partial t}\frac{1}{\left(I_i(t)\right)^b} = \frac{1}{1-b}\frac{\partial W_i(t)}{\partial t}$$

After the change of variable, equation (3) becomes

$$\frac{\partial W_i(t)}{\partial t} - (\rho+\delta)W_i(t) = -\lambda_i A^{\alpha/2} b, \tag{17}$$

which is a first-order nonhomogeneous linear differential equation. The solution to (17) is given by

$$W_i(t) = C_i e^{(\rho+\delta)t} + \frac{\lambda_i A^{\alpha/2} b}{\rho+\delta}, \tag{18}$$

where C_i is a constant. At the end of individual's working life investment in human capital must be zero so

$$0 = C_i e^{(\rho+\delta)T} + \frac{\lambda_i A^{\alpha/2} b}{\rho+\delta}.$$

Solving for C_i we have that

$$C_i = -\frac{\lambda_i A^{\alpha/2} b}{\rho+\delta} e^{-(\rho+\delta)T}. \tag{19}$$

Substituting (19) into (18) we have that

$$W_i(t) = \frac{\lambda_i A^{\alpha/2} b}{\rho+\delta}\left(1 - e^{(\rho+\delta)(T-t)}\right),$$

or

$$I_i(t) = \left(\frac{\lambda_i A^{\alpha/2} b}{\rho+\delta}\right)^{\frac{1}{1-b}} \left(1 - e^{(\rho+\delta)(T-t)}\right)^{\frac{1}{1-b}},$$

which is equation (3). *Q.E.D.*

Derivation of Equation (6) Rearranging (5) we have that

$$\frac{\partial K_i(t)}{\partial t} + \delta K_i(t) = \frac{1}{2}\frac{A^\alpha \lambda_i}{\rho+\delta}\left(1 - e^{-(\rho+\delta)(T-t)}\right), \quad i = 1, 2.$$

The solution to this differential equation is given by

$$\begin{aligned} K_i(t) &= e^{-\delta t}\left[C + \frac{1}{2}\frac{A^\alpha \lambda_i}{\rho+\delta}\int \left(1 - e^{-(\rho+\delta)(T-t)}\right) e^{\delta t} dt\right] \\ &= C_i e^{-\delta t} + \frac{1}{2}\frac{A^\alpha \lambda_i}{(\rho+\delta)\delta} - \frac{1}{2}\frac{A^\alpha \lambda_i}{(\rho+\delta)(\rho+2\delta)} e^{-(\rho+\delta)(T-t)}, \end{aligned} \tag{20}$$

where C_i is a constant. At the start of an individual's working life the stock of skill i is given by $K_i(0)$ so

$$K_i(0) = C_i + \frac{1}{2}\frac{A^\alpha \lambda_i}{(\rho+\delta)\delta} - \frac{1}{2}\frac{A^\alpha \lambda_i}{(\rho+\delta)(\rho+2\delta)} e^{-(\rho+\delta)T}.$$

Solving for C_i we have that

$$C_i = K_i(0) - \frac{1}{2}\frac{A^\alpha \lambda_i}{(\rho+\delta)\delta} + \frac{1}{2}\frac{A^\alpha \lambda_i}{(\rho+\delta)(\rho+2\delta)} e^{-(\rho+\delta)T} \tag{21}$$

Substituting (21) into (20) we have that

$$K_i(t) = K_i(0)e^{-\delta t} + \frac{A^\alpha \lambda_i}{2(\rho+\delta)\delta}\left[1 - e^{-\delta t} - \frac{\delta e^{-(\rho+\delta)(T-t)}}{\rho+2\delta}\left(1 - e^{-(\rho+2\delta)t}\right)\right],$$

which is equation (6). *Q.E.D.*

Proof of Lemma 1 Setting $t = 0$ in (7) we have that $\omega(0) = 0$. Setting $t = T$ in (7) we have that

$$\text{sign}\,\omega(T) = \text{sign}\left(1 - e^{-\delta T} - \frac{\delta}{\rho+2\delta}(1 - e^{-(\rho+2\delta)T})\right). \tag{22}$$

From (22) we see that the sign of $\omega(T)$ is positive if

$$\frac{\delta}{\rho+2\delta} < \frac{1-e^{-\delta T}}{1-e^{-(\rho+2\delta)T}}. \tag{23}$$

Let us now show that inequality (23) is valid. Let $\rho = k\delta$ with $k > 0$. Substituting $\rho = k\delta$ into (23) we have $\frac{1}{k+2} < \frac{1-e^{-\delta T}}{1-e^{-(k+2)\delta T}}$.Now, let $y = \delta T$. We have that $\frac{1-e^{-y}}{1-e^{-(k+2)y}}$ is increasing with y and that $\lim_{y\to 0}\frac{1-e^{-y}}{1-e^{-(k+2)y}} = \frac{1}{k+2}$ and $\lim_{y\to\infty}\frac{1-e^{-y}}{1-e^{-(k+2)y}} = 1$. This implies inequality (23) is valid and so $\omega(T) > 0$. Taking the first derivative of $\omega(t)$ we obtain

$$\frac{d\omega}{dt} = \frac{1}{2(\rho+\delta)}\left[e^{-\delta t} - \frac{\delta}{\rho+2\delta}e^{-(\rho+\delta)T-\delta t} - \frac{\rho+\delta}{\rho+2\delta}e^{-(\rho+\delta)(T-t)}\right]. \tag{24}$$

The second derivative of $\omega(t)$ is given by

$$\frac{d^2\omega}{dt^2} = \frac{1}{2(\rho+\delta)}\left[-\delta e^{-\delta t} - \frac{e^{-(\rho+\delta)T}}{\rho+2\delta}\left[(\rho+\delta)^2 e^{(\rho+\delta)t} - \delta^2 e^{-\delta t}\right]\right]. \tag{25}$$

Since $\delta^2 e^{-\delta t} < (\rho+\delta)^2 e^{(\rho+\delta)t}$ the term inside square brackets in (25) is negative and so $d^2\omega/dt^2 < 0$. Thus, $\omega(t)$ is a concave function. From (24) we have that

$$\text{sign}\left(d\omega/dt|_{t=0}\right) = \text{sign}\left(1 - e^{-(\rho+\delta)T}\right).$$

Since $e^{-(\rho+\delta)T} < 1$ we have that $d\omega/dt|_{t=0} > 0$. From (24) we also have that

$$\begin{aligned}\text{sign}\left(d\omega/dt|_{t=T}\right) &= \text{sign}\left(e^{-\delta T} - \frac{\delta}{\rho+2\delta}e^{-(\rho+2\delta)T} - \frac{\rho+\delta}{\rho+2\delta}\right) \\ &= \text{sign}\left(\left(\rho+2\delta-\delta e^{-(\rho+\delta)T}\right)e^{-\delta T} - \rho + \delta\right) \qquad (26)\end{aligned}$$

From (26) we see that the sign of $d\omega/dt|_{t=T}$ is negative if

$$\left(\rho + 2\delta - \delta e^{-(\rho+\delta)T}\right) e^{-\delta T} < \rho + \delta. \tag{27}$$

We will now show that inequality (27) is valid. Rearranging (27) we have

$$\frac{\delta}{\rho+\delta} e^{-\delta T} < \frac{1 - e^{-\delta T}}{1 - e^{-(\rho+\delta)T}}.$$

Since $e^{-\delta T} < 1$ we have that $\frac{\delta}{\rho+\delta} e^{-\delta T} < \frac{\delta}{\rho+\delta}$.But, we know that $\frac{\delta}{\rho+\delta} < \frac{1-e^{-\delta T}}{1-e^{-(\rho+\delta)T}}$. These two inequalities imply that inequality (27) is valid and so $d\omega/dt|_{t=T} < 0$. The fact that $d\omega/dt|_{t=0} > 0$, $d\omega/dt|_{t=T} < 0$, together with the fact that $\omega(t)$ is a concave function imply that $\omega(t)$ attains its maximum at t^*, with $t^* \in (0, T)$. *Q.E.D.*

Proof of Proposition 1 The change in the expected ability gap over time is completely determined by the change in $\omega(t)$ over time. Thus, Lemma 1 implies that the expected ability gap is increasing with t for $0 < t < t^*$ and decreasing with t for $t^* < t < T$, where $t^* = \arg\max \omega(t)$. *Q.E.D.*

Proof of Proposition 2 The proof is a direct application of Proposition 9 in Santos-Pinto and Sobel (2005). If $\alpha \in (0,1)$ then $D^*(t; K(0), A, \lambda)$ is concave in A and so a mean preserving spread in the distribution of ability to produce human capital decreases $E_A D^*(t; K(0), A, \lambda)$. *Q.E.D.*

Proof of Proposition 3 From (10) see that $K_i(0) \geq \bar{K}_i(0)$ implies that the first two terms in (10) are nonnegative. We also see that $A^\alpha \left[\lambda^2 + (1-\lambda^2)\right] - E(A^\alpha)\frac{1}{2} < 0$ implies that the third term in (10) is negative. For $t \in (0, t^*)$, an increase in t increases the contribution of the third term and reduces the contribution of the first two terms to the individual's ability gap. *Q.E.D.*

Proof of Proposition 4 The derivative of $\omega(t)$ with respect to ρ is equal to

$$\begin{aligned}\frac{d\omega(t)}{d\rho} = &-\frac{\omega(t)}{(\rho+\delta)} - \frac{e^{-(\rho+\delta)(T-t)} - e^{-(\rho+\delta)T-\delta t}}{2(\rho+\delta)(\rho+2\delta)^2} \\ &- \frac{(T-t)\, e^{-(\rho+\delta)(T-t)} - Te^{-(\rho+\delta)T-\delta t}}{2(\rho+\delta)\,(\rho+2\delta)}.\end{aligned}$$

By Lemma 1 $\omega(t)$ is nonnegative. The numerator in the second term is nonnegative. The numerator in the third term is also nonnegative since $(T-t)/T \geq e^{-(\rho+2\delta)t}$ for $t \in [0, T)$. We also have that

$$\left.\frac{d\omega(t)}{d\rho}\right|_{t=T} = -\frac{\omega(T)}{(\rho+\delta)} - \frac{1 - \left[1 + (\rho+2\delta)\, Te^{-(\rho+2\delta)T}\right]}{2(\rho+\delta)(\rho+2\delta)^2} \tag{28}$$

The fact that $1/(1+z) > e^{-z}$ for $z > 0$ implies that the numerator in the second term in (28) is positive. So, $d\omega(t)/d\rho \leq 0$ for $t \in [0, T]$. *Q.E.D.*

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